Solar Collectors: Solar energy collectors are special kinds of heat exchangers that transform solar radiation energy to internal energy of the transport medium. The major component of any solar system is the solar collector. This is a device that absorbs the incoming solar radiation, converts it into heat, and transfers the heat to a fluid (usually air, water, or oil) flowing through the collector. The basic parameter to consider is the collector thermal efficiency. This is defined as the ratio of the useful energy delivered to the energy incident on the collector aperture. The incident solar flux consists of direct and diffuse radiation. While flat-plate collectors can collect both, concentrating collectors can utilize direct radiation only if the **concentration ratio** is greater than (10).

1. Flat-Plate Collectors:

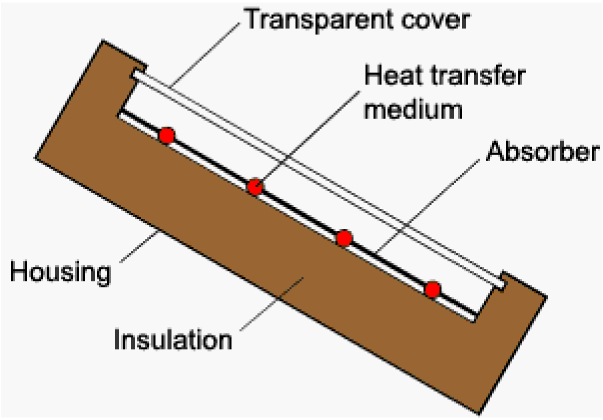

Figure (1): flat plate collector construction.

2. Collector Energy Losses:

When a certain amount of solar radiation falls on the surface of a collector, most of it is absorbed and delivered to the transport fluid, and it is carried away as useful energy. However, as in all thermal systems, heat losses to the environment by various modes of heat transfer are inevitable. The thermal network for a single- cover, flat-plate collector in terms of conduction, convection, and radiation is shown in Figure (2-a) and in terms of the resistance between plates in Figure (2-b). The temperature of the plate is (T_p), the collector back temperature is (T_b), and the absorbed solar radiation is (S**).** In a simplified way, the various thermal losses from the collector can be combined into a simple resistance, (R_L), as shown in Figure (4-c), so that the energy losses from the collector can be written as

$$Q_{loss} = \left((T_p - T_a)/R_L\right) = U_L A_c \left(T_p - T_a\right) \ldots (1)$$

U_L= overall heat loss coefficient based on collector area (A_c)(W/m^2 -K).
T_p= plate temperature (°C).

The overall heat loss coefficient is a complicated function of the collector construction and its operating conditions, given by the following expression:

$$U_L = U_t + U_b + U_e \ldots (2)$$

$\boldsymbol{U_t}$ = top loss coefficient (W/m^2 -K). $\boldsymbol{U_b}$ = bottom heat loss coefficient (W/m^2 -K).
$\boldsymbol{U_e}$ = heat loss coefficient form the collector edges (W/m^2 -K).

Therefore, (UL) is the heat transfer resistance from the absorber plate to the ambient air.

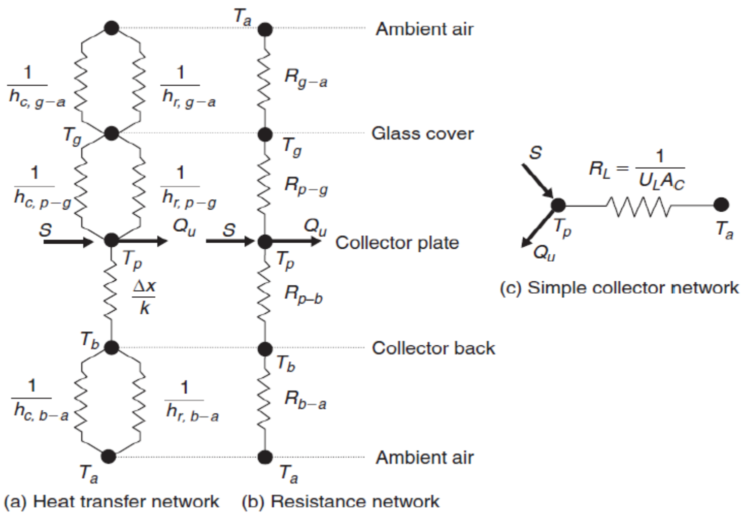

Fig (2): Thermal network for a single cover collector in terms of (a) conduction, convection, and radiation; (b) resistance between plates; and (c) a simple collector network.

Under steady-state conditions, the heat transfer from the absorber plate to the glass cover is the same as the energy lost from the glass cover to ambient. The heat transfer upward from the absorber plate at temperature **(T_p)** to the glass cover at **(T_g)** and from the glass, cover at **(T_g)** to ambient at **(T_a)** is by convection and infrared radiation. Therefore, the heat loss from absorber plate to glass is given by

$$Q_{t,p-g} = A_c h_{c,p-g}\left(T_p - T_g\right) + \frac{A_c \sigma\left(T_p^4 - T_g^4\right)}{(1/\varepsilon_p)+(1/\varepsilon_g)-1} \ldots (3)$$

$\boldsymbol{A_c}$ = Collector area (m^2).

$\boldsymbol{h_{c,p-g}}$ =Coefficient of Convection heat transfer between the absorber plate and glass cover (W/m^2 -K).

$\boldsymbol{\varepsilon_p}$ =Infrared emissivity of absorber plate.

ε_g =Infrared emissivity of glass cover.

For tilt angles up to **60°**, the convective heat transfer coefficient, $h_{c,p-g}$, is given for collector inclination **(θ)** in degrees:

$$N_u = \frac{h_{c,p-g} \times L}{k} = 1 + 1.446\left[1 - \frac{1708}{R_a \times \cos(\theta)}\right]^{+}\left\{1 - \frac{1708\left(\sin(1.8\theta)^{1.6}\right)}{R_a \times \cos(\theta)}\right\} + \left[\left[\frac{R_a \times \cos(\theta)}{5830}\right]^{0.333} - 1\right]^{+} \ldots (4)$$

Where the plus sign represents positive values only. The Rayleigh value, **(R_a)**, is given by

$$R_a = \frac{g\, P_r\, \acute{\beta}}{v^2}\left(T_p - T_g\right)L^3 \ldots (5)$$

g = gravitational constant, =9.81 m^2 /s.

L = absorber to glass cover distance (m).

$\boldsymbol{P_r}$ = Prandtl number.

ν = kinetic viscosity (m^2 /s).

$\boldsymbol{\acute{\beta}}$= volumetric coefficient of expansion; for ideal gas, $\boldsymbol{\acute{\beta}}$= 1/ T.

The fluid properties in equation (5) are evaluated at the mean gap temperature **($T_p + T_g$)/2.**

The radiation heat transfer coefficient is given by

$$h_{r,p-g} = \frac{\sigma\left(T_p + T_g\right)\left(T_p^2 + T_g^2\right)}{(1/\varepsilon_p)+(1/\varepsilon_g)-1} \ldots (6)$$

Consequently, equation (3) becomes

$$Q_{t,p-g} = A_c(h_{c,p-g} + h_{r,p-g})(T_p - T_g) = \frac{(T_p - T_g)}{R_{p-g}} \ldots (7)$$

$$R_{p-g} = \frac{1}{A_c(h_{c,p-g} + h_{r,p-g})} \ldots (8)$$

Similarly, the heat loss from glass cover to ambient is given by

$$Q_{t,g-a} = A_c(h_{c,g-a} + h_{r,g-a})(T_g - T_a) = \frac{(T_g - T_a)}{R_{g-a}} \ldots (9)$$

$h_{c,g-a}$ = Coefficient of Convection heat transfer between the glass cover and ambient due to wind (W/m^2-K).

$h_{r,g-a}$ = Coefficient of radiation heat transferbetween the glass cover and ambient (W/m^2-K).

$$h_{r,g-a} = \varepsilon_g \sigma (T_g + T_a)(T_g^2 + T_a^2) \ldots (10)$$

$$R_{g-a} = \frac{1}{A_c(h_{c,g-a} + h_{r,g-a})} \ldots (11)$$

Since resistances and are in series, their resultant is given by

$$R_t = R_{p-g} + R_{g-a} = \frac{1}{U_t A_c} \ldots (12)$$

$$Q_t = \frac{(T_p - T_a)}{R_t} = U_t A_c (T_p - T_a) \ldots (13)$$

In some cases, collectors are constructed with two glass covers in an attempt to lower heat losses. By following a similar analysis, the heat transfer from the lower glass at **(T_{g2})** to the upper glass at **(T_{g1})** is given by

$$Q_{t,g2-g1} = A_c(h_{c,g2-g1} + h_{r,g2-g1})(T_{g2} - T_{g1}) = \frac{(T_{g2} - T_{g1})}{R_{g2-g1}} \ldots (14)$$

$h_{c,g2-g1}$ = Coefficient of Convection heat transfer between the two glasses covers (W/m^2-K).

$h_{r,g2-g1}$ =Coefficient of radiation heat transfer between the two glasses covers (W/m^2-K).

The Coefficient of radiation heat transfer is given by

$$h_{r,g2-g1} = \frac{\sigma(T_{g2} + T_{g1})(T_{g2}^2 + T_{g1}^2)}{(1/\varepsilon_{g2}) + (1/\varepsilon_{g1}) - 1} \ldots (15)$$

Where $\varepsilon_{g2}$and ε_{g1} are the infrared emissivity of the top and bottom glass covers.

$$R_{g2-g1} = [(1/A_c(h_{c,g2-g1} + h_{r,g2-g1}))] \ldots (16)$$

In the case of collectors with two covers, equation (16) is added on the resistance values in equation (12).

The top heat loss coefficient (U_t), since the air properties are functions of operating temperature, is given by the following empirical equation with sufficient accuracy for design purposes

$$U_t = \frac{1}{\dfrac{N_g}{\dfrac{C}{T_p}\left[\dfrac{T_p - T_a}{N_g + f}\right]^{0.33} + \dfrac{1}{h_w}}} + \frac{\sigma\left(T_p^2 + T_a^2\right)\left(T_p + T_a\right)}{\dfrac{1}{\varepsilon_{p+0.05N_g(1-\varepsilon_p)}} + \dfrac{2N_g + f - 1}{\varepsilon_g} - N_g} \quad \ldots (17)$$

$$f = (1 - 0.04h_w + 0.0005h_w^2)(1 + 0.091N_g) \ldots (18)$$

$$c = 365.9(1 - 0.00883\beta + 0.000129\beta^2) \ldots (19)$$

$$h_w = \frac{8.6V^{0.6}}{L^{0.4}} \ldots (20)$$

It should be noted that, for the wind heat transfer coefficient, could be used from equation (20).
The energy loss from the bottom of the collector is first conducted through the insulation and then by a combined convection and infrared radiation transfer to the surrounding ambient air. Because the temperature of the bottom part of the casing is low, the radiation term ($h_{r,b-a}$) can be neglected; thus the energy loss is given by

$$U_b = \frac{1}{(t_b/k_b) + (1/h_{c,b-a})} \ldots (21)$$

$\boldsymbol{t_b}$ = thickness of back insulation (m). $\boldsymbol{k_b}$ = conductivity of back insulation (W/m-K).
$\boldsymbol{h_{c,b-a}}$ =convection heat loss coefficient from back to ambient (W/m^2 -K).

In a similar way, the heat transfer coefficient for the heat loss from the collector edges can be obtained from

$$U_e = \left[1/\langle (t_e/k_e) + (1/h_{c,e-a})\rangle\right] \ldots (22)$$

$\mathbf{t_e}$ = thickness of edge insulation (m). $\mathbf{k_e}$ = conductivity of edge insulation (W/m-K).
$\mathbf{h_{c,e-a}}$ =convection heat loss coefficient from edge to ambient (W/m^2 -K).

Example: Estimate the top heat loss coefficient of a collector that has the following specifications:

- Collector area = 2 m^2 (1× 2 m).
- Collector slope = 35°.
- Number of glass covers = 2.
- Thickness of each glass cover = 4 mm.
- Thickness of absorbing plate = 0.5 mm.
- Space between glass covers = 20 mm.
- Space between inner glass cover and absorber = 40 mm.
- Thickness of back insulation = 50 mm.
- Back insulation thermal conductivity = 0.05 W/m-K.
- Mean absorber temperature, T_p = 80° C = 353 K.
- Ambient air temperature, T_a = 15° C = 288 K.
- Absorber plate emissivity, ε_p = 0.10
- Glass emissivity, ε_g = 0.88
- Wind velocity = 2.5 m/s.

Solution:

$$(h_{c,p-g2} + h_{r,p-g2})(T_p - T_{g2}) = (h_{c,g2-g1} + h_{r,g2-g1})(T_{g2} - T_{g1})$$
$$= (h_{c,g1-a} + h_{r,g1-a})(T_{g1} - T_a)$$

Assuming that T_{g1}= 23.8° C (296.8 K) and T_{g2}= 41.7° C (314.7 K).

$$h_{r,p-g2} = \frac{\sigma(T_p + T_{g2})(T_p^2 + T_{g2}^2)}{(1/\varepsilon_p) + (1/\varepsilon_{g2}) - 1}$$
$$= \frac{(5.67 \times 10^{-8})(353 + 314.7)(353^2 + 314.7^2)}{(1/0.10) + (1/0.88) - 1}$$
$$= 0.835\ W/m^2 - K$$

$$h_{r,g2-g1} = \frac{\sigma(T_{g2}+T_{g1})(T_{g2}^2+T_{g1}^2)}{(1/\varepsilon_{g2})+(1/\varepsilon_{g1})-1}$$
$$= \frac{(5.67\times10^{-8})(314.7+296.8)(314.7^2+296.8^2)}{(1/0.88)+(1/0.88)-1}$$
$$= 5.098\ W/m^2-K$$

$$h_{r,g1-a} = \varepsilon_{g1}\sigma(T_{g1}+T_a)(T_{g1}^2+T_a^2) = 0.88(296.8+288)(296.8^2+288^2) = 4.991\ \mathrm{W/m^2-K}$$

$$for\ \frac{1}{2}(T_p+T_{g2}) = \frac{1}{2}(353+314.7) = 333.85\ K,$$
$$\nu = 19.51\times10^{-6}\ m^2/s\ ;\ Pr = 0.701\ ;\ k = 0.0288\ W/m-K.$$
$$for\ \frac{1}{2}(T_{g2}+T_{g1}) = \frac{1}{2}(314.7+296.8) = 305.75\ K,$$
$$\nu = 17.26\times10^{-6}\ m^2/s\ ;\ Pr = 0.707\ ;\ k = 0.0267\ W/m-K.$$

By using these properties, and by noting that $\acute{\beta}$= 1/ T. the Rayleigh number, **(Ra),** can be obtained

For $(h_{c,p-g2})$

$$Ra = \frac{gPr\beta'}{\nu^2}(T_p-T_{g2})L^3 = \frac{9.81\times0.701\times(353-314.7)\times0.04^3}{333.85\times(19.51\times10^{-6})^2}$$
$$= 132{,}648$$

For $(h_{c,g2-g1})$

$$Ra = \frac{gPr\beta'}{\nu^2}(T_{g2}-T_{g1})L^3 = \frac{9.81\times0.701\times(314.7-296.8)\times0.02^3}{305.75\times(17.26\times10^{-6})^2}$$
$$= 10{,}904$$

$$for\ h_{c,p-g2}$$

$$h_{c,p-g2} = \frac{k}{L}\left\{1+1.446\left[1-\frac{1708}{Ra\times\cos(\theta)}\right]^+\left[1-\frac{1708(\sin(1.8\times\theta)^{1.6}}{Ra\times\cos(\theta)}\right] + \left\langle\left[\left[\frac{Ra\times\cos(\theta)}{5830}\right]^{0.333}-1\right]^+\right\rangle\right\}$$

$$h_{c,p-g2} = \frac{0.0288}{0.04}\left\{1+1.446\left[1-\frac{1708}{132{,}648\times\cos(35)}\right]^+\left[1-\frac{1708(\sin(1.8\times35)^{1.6}}{132{,}648\times\cos(35)}\right] + \left\langle\left[\left[\frac{132{,}648\times\cos(35)}{5830}\right]^{0.333}-1\right]^+\right\rangle\right\} = 2.918\ W/m^2-K$$

$$for\ h_{c,g2-g1}$$

$$h_{c,g2-g1} = \frac{k}{L}\left\{1+1.446\left[1-\frac{1708}{Ra\times\cos(\theta)}\right]^+\left[1-\frac{1708(\sin(1.8\times\theta)^{1.6}}{Ra\times\cos(\theta)}\right] + \left\langle\left[\left[\frac{Ra\times\cos(\theta)}{5830}\right]^{0.333}-1\right]^+\right\rangle\right\}$$

$$h_{c,p-g2} = \frac{0.0267}{0.02}\left\{1+1.446\left[1-\frac{1708}{10{,}904\times\cos(35)}\right]^+\left[1-\frac{1708(\sin(1.8\times35)^{1.6}}{10{,}904\times\cos(35)}\right] + \left\langle\left[\left[\frac{10{,}904\times\cos(35)}{5830}\right]^{0.333}-1\right]^+\right\rangle\right\} = 2.852\ W/m^2-K$$

The convection heat transfer coefficient from glass to ambient is the wind loss coefficient given by equation (20). In this equation, the characteristic length is the length of the collector, equal to 2 m. Therefore,

$$h_{c,g1-a} = h_w = \frac{8.6V^{0.6}}{L^{0.4}} = \frac{8.6(2.5)^{0.6}}{2^{0.4}} = 11.294\ W/m^2 - K$$

$$U_t = \left(\frac{1}{h_{c,p-g2} + h_{r,p-g2}} + \frac{1}{h_{c,g2-g1} + h_{r,g2-g1}} + \frac{1}{h_{c,g1-a} + h_{r,g1-a}}\right)^{-1}$$

$$U_t = \left(\frac{1}{2.918 + 0.835} + \frac{1}{2.852 + 5.098} + \frac{1}{11.294 + 4.991}\right)^{-1} = 2.204\ W/m^2 - K$$

<u>Example</u>: repeat a previous example using the empirical equation (42) and compare the results.

<u>Solution</u>: First, the constant parameters are estimated. The value of h_w is already estimated in a previous example and is equal to 11.294 W/m²-K.

$$f = (1 - 0.04h_w + 0.0005h_w^2)(1 + 0.091N_g)$$
$$f = (1 - 0.04 \times 11.294 + 0.0005 \times 11.294^2)(1 + 0.091 \times 2) = 0.723$$

$$c = 365.9(1 - 0.00883\beta + 0.000129\beta^2)$$
$$c = 365.9(1 - 0.00883 \times 35 + 0.000129 \times 35^2) = 311$$

$$U_t = \frac{1}{\dfrac{N_g}{\dfrac{c}{T_p}\left[\dfrac{T_p - T_a}{N_g + f}\right]^{0.33} + \dfrac{1}{h_w}}} + \frac{\sigma(T_p^2 + T_a^2)(T_p + T_a)}{\dfrac{1}{\varepsilon_{p} + 0.05N_g(1 - \varepsilon_p)} + \dfrac{2N_g + f - 1}{\varepsilon_g} - N_g}$$

$$U_t = \frac{1}{\dfrac{2}{\dfrac{311}{353}\left[\dfrac{353 - 288}{2 + 0.732}\right]^{0.33} + \dfrac{1}{11.294}}} + \frac{5.67 \times 10^{-8}(353^2 + 288^2)(353 + 288)}{\dfrac{1}{0.1 + 0.05 \times 2(1 - 0.1)} + \dfrac{(2 \times 2) + 0.723 - 1}{0.88} - 2} = 2.306\ W/m^2 - K$$

<u>3. Temperature Distribution between the Tubes and Collector Efficiency Factor:</u>

Under steady-state conditions, the rate of useful heat delivered by a solar collector is equal to the rate of energy absorbed by the heat transfer fluid minus the direct or indirect heat losses from the surface to the surroundings figure (3).

The thermal energy lost from the collector to the surroundings by conduction, convection, and the product of the overall heat loss coefficient, (U_L), times the difference between the plate temperature, (T_p), and the ambient temperature, (T_a), represents infrared radiation.

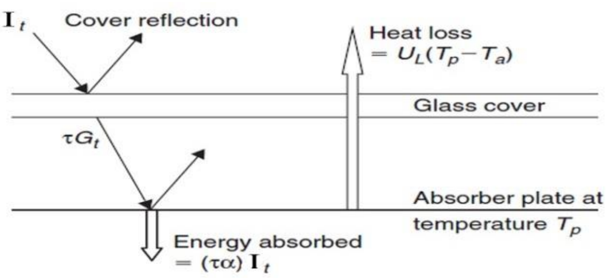

Figure (3): Radiation input and heat loss from a flat-plate collector.

Therefore, in a steady state, the rate of useful energy collected from a collector of area Ac can be obtained from

$$Q_u = A_c\left[I_t(\tau\alpha) - U_L\left(T_p - T_a\right)\right] = \dot{m}c_p[T_o - T_i] \ldots (23)$$

The collector efficiency factor can be calculated by considering the temperature distribution between two pipes of the collector absorber and assuming that the temperature gradient in the flow direction is negligible. This analysis can be performed by considering the sheet-tube configuration shown in Figure (4-a), where the distance between the tubes is (W), the tube diameter is (D), and the sheet thickness is (δ). Since the sheet metal is usually made from copper or aluminum, which are good conductors of heat, the temperature gradient through the sheet is negligible; therefore, the region between the centerline separating the tubes and the tube base can be considered as a classical fin problem.

The energy conducted to the region of the tube per unit length in the flow direction can be found by evaluating the Fourier's law at the fin base

$$q'_{fin} = (W - D)[S - U_L(T_b - T_a)]\frac{\tanh[m(W-D)/2]}{m(W-D)/2} \ldots (24)$$

$$q'_{fin} = (W - D)F[S - U_L(T_b - T_a)] \ldots (25)$$

Where factor (F) in equation (25) is the **standard fin efficiency** for straight fins with a rectangular profile, obtained

$$F = \frac{\tanh[m(W-D)/2]}{m(W-D)/2} \ldots (26), \text{ Where } m = \sqrt{U_L/k\delta}$$

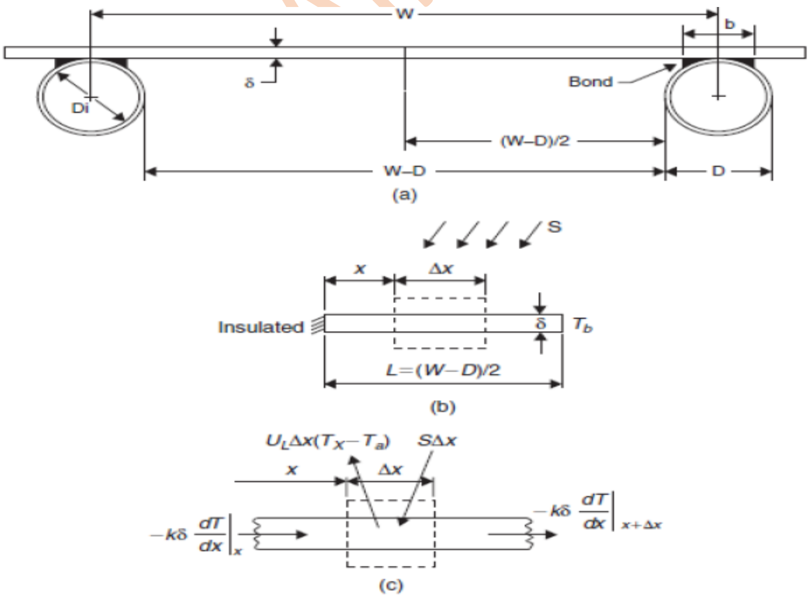

Figure (4): Flat-plate sheet and tube configuration. (a) Schematic diagram. (b) Energy balance for the fin element. (c) Energy balance for the tube element.

The useful gain of the collector also includes the energy collected above the tube region. This is given by

$$q'_{tube} = D[S - U_L(T_b - T_a)] \ldots (27)$$

Accordingly, the useful energy gain per unit length in the direction of the fluid flow is

$$q'_u = q'_{fin} + q'_{tube} = [(W - D)F + D][S - U_L(T_b - T_a)] \ldots (28)$$

This energy ultimately must be transferred to the fluid, which can be expressed in terms of two resistances as

$$q'_u = \frac{T_b - T_f}{(1/\pi D_i h_{fi}) + (1/c_b)} \ldots (29)$$

h_{fi} = Heat transfer coefficient between the fluid and the tube wall, c_b = is the bond conductance.

$$c_b = \langle k_b b/\gamma \rangle \ldots (30)$$

k_b = The bond thermal conductivity, γ = the average bond thickness, b =the bond width.

Therefore, the useful gain

$$q'_u = [WF'][S - U_L(T_f - T_a)] \ldots (31)$$

Where F′ is the collector efficiency factor, given by

$$F' = \frac{(1/U_L)}{W\left\{\frac{1}{U_L[D+(W-D)F]} + \frac{1}{c_b} + \frac{1}{\pi D_i h_{fi}}\right\}} \ldots (32)$$

It should be noted that the denominator of equation (32) is the heat transfer resistance from the fluid to the ambient air. This resistance can be represented as $(1/U_o)$ .Therefore, another interpretation of F' is

$$F' = (U_o/U_L) \ldots (33)$$

<u>Example</u>: For a collector having the following characteristics and ignoring the bond resistance, calculate the fin efficiency and the collector efficiency factor:

Overall loss coefficient = 6.9 W/m^2 °C.
Tube spacing = 120 mm.
Tube outside diameter = 15 mm.
Tube inside diameter = 13.5 mm.
Plate thickness = 0.4 mm.
Plate material = copper.
Heat transfer coefficient inside the tubes = 320 W/m^2 -°C.

<u>Solution</u>: From Table for copper, k = 385 W/m-°C.

$$m = \sqrt{U_L/k\delta} = \sqrt{6.9/385 \times 0.0004} = 6.69\, m^{-1}$$

$$F = \frac{\tanh[m\,(W - D)/2]}{m\,(W - D)/2} = \frac{\tanh[6.69\,(0.12 - 0.015)/2]}{6.69\,(0.12 - 0.015)/2} = 0.961$$

$$F' = \frac{(1/U_L)}{W\left\{\frac{1}{U_L[D+(W-D)F]} + \frac{1}{c_b} + \frac{1}{\pi D_i h_{fi}}\right\}} = \frac{(1/6.9)}{0.12\left\{\frac{1}{6.9[0.015+(0.12-0.015)0.961]} + 0 + \frac{1}{\pi \times 0.0135 \times 320}\right\}} = 0.912$$

<u>4. Heat Removal Factor, Flow Factor, and Thermal Efficiency</u>:

Consider an infinitesimal length δ_y of the tube as shown in figure (5). The useful energy delivered to the fluid is δ_y. It is usually desirable to express the collector total useful energy gain in terms of the fluid inlet temperature.

To do this the collector heat removal factor needs to be used.

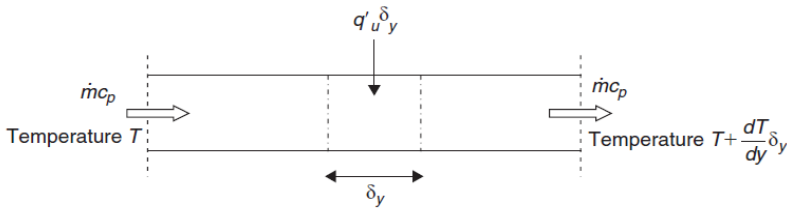

Figure (5): Energy flow through an element of riser tube.

Heat removal factor expressed symbolically,

$$F_R = \frac{m^{\circ} c_p}{A_c U_L}\left[1 - exp\left\{-\frac{A_c U_L F'}{m^{\circ} c_p}\right\}\right] \dots (34)$$

Another parameter usually used in the analysis of collectors is the flow factor. This is defined as the ratio of (F_R) to (F'), given by

$$F'' = \frac{F_R}{F'} = \frac{m^{\circ} c_p}{A_c U_L}\left[1 - exp\left\{-\frac{A_c U_L F'}{m^{\circ} c_p}\right\}\right] \dots (35)$$

As shown in equation (35), the collector flow factor is a function of only a single variable, the dimensionless collector capacitance rate$m^{\circ} c_p / A_c U_L F'$, shown in figure (5).

$$Q_u = A_c F_R [I_t(\tau\alpha) - U_L(T_i - T_a)] \dots (36)$$

The critical radiation level can also be defined. This is the radiation level where the absorbed solar radiation and loss term are equal. This is obtained by setting the term in the right-hand side of equation (61) equal to 0 (or Q_u = 0). Therefore, the critical radiation level, (I_{tc}), is given by

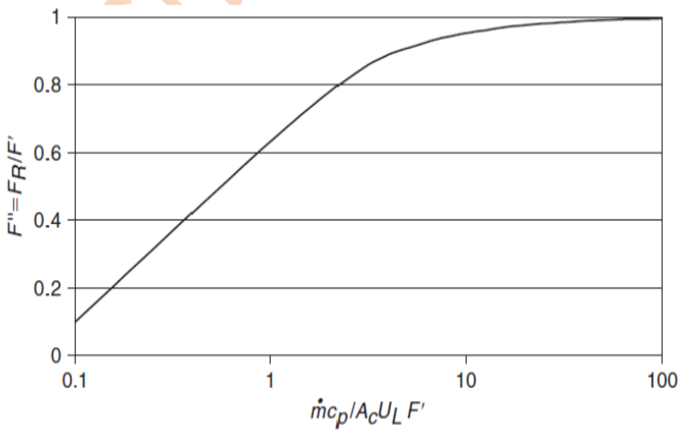

Fig (6): Collector flow factor as a function of the dimensionless capacitance rate.

$$I_{tc} = \frac{U_L(T_i - T_a)}{(\tau\alpha)} \dots (37)$$

Finally, the collector efficiency can be obtained by dividing Q_u, equation (36), by ($I_t A_c$). Therefore,

$$\eta = F_R\left[(\tau\alpha) - \frac{U_L(T_i - T_a)}{I_t}\right] \dots (38)$$

Example:
For the collector outlined in previous example, calculate the useful energy and the efficiency if collector area is (4 m^2), flow rate is **0.06** kg/s, **(τα) = 0.8**, the global solar radiation for **1 h** is **2.88 MJ/m^2**, and the collector operates at a temperature difference of **5°C.**

Solution:
The dimensionless collector capacitance rate is

$$\frac{\dot{m}c_p}{A_c U_L F'} = \frac{0.06 * 4180}{4 * 6.9 * 0.91} = 9.99$$

$$F'' = 9.99\left[1 - exp\left\{-\frac{1}{9.99}\right\}\right] = 0.952$$

$$F_R = F' \times F'' = 0.91 \times 0.952 = 0.866,$$

$$Q_u = A_c F_R[I_t(\tau\alpha) - U_L(T_i - T_a)]$$
$$= 4 \times 0.866\left[2.88 \times 0.8 - (6.9 \times 5 \times \frac{3600}{1000000})\right] = 7.55\ MJ$$

$$\eta = (Q_u/A_c I_t) = \frac{7.55}{4 \times 2.88} \rightarrow \eta = 65.5\%$$

5. Thermal analysis of air collectors:

A schematic diagram of a typical air-heating flat-plate solar collector is shown in Figure (7). The air passage is a narrow duct with the surface of the absorber plate serving as the top cover. The thermal analysis presented so far applies equally well here, except for the fin efficiency and the bond resistance.

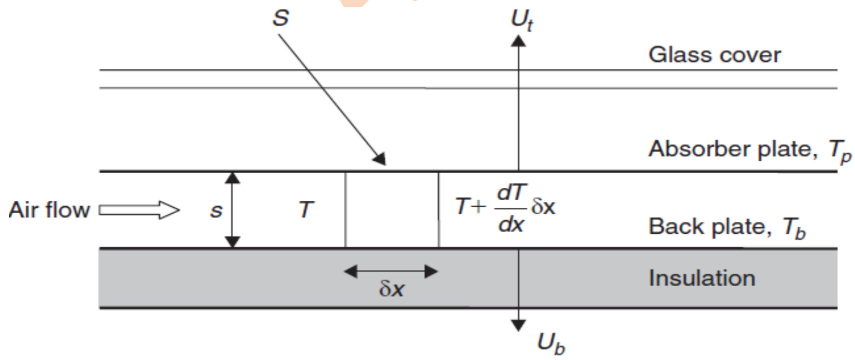

Figure (7): Schematic diagram of an air-heating collector.

An energy balance on the absorber plate of area (1 ×δx) gives

$$S(\delta_x) = U_t(\delta_x)(T_p - T_a) + h_{c,p-a}(\delta_x)(T_p - T) + h_{r,p-b}(\delta_x)(T_p - T_b) \ldots (39)$$

$h_{c,p-a}$ = Convection heat transfer coefficient from absorber plate to air (W/m^2 -K).

$h_{r,p-b}$ = Radiation heat transfer coefficient from absorber plate to back plate (W/m^2 -K), which can be obtained from;

$$h_{r,p-b} = \frac{\sigma(T_p+T_b)(T_p^2+T_b^2)}{(1/\varepsilon_p)+(1/\varepsilon_b)-1} \ldots (40)$$

An energy balance of the air stream volume (s × 1 ×δx) gives

$$\left[\frac{m^{\circ}}{W}\right] c_p \left[\frac{dT}{dx}\delta_x\right] = h_{c,p-a}(\delta_x)\left(T_p - T\right) + h_{c,b-a}(\delta_x)(T_b - T) \ \ldots (41)$$

$h_{c,b-a}$ = Convection heat transfer coefficient from the back plate to air (W/m^2 -K).
An energy balance on the back plate area (1 ×δx) gives

$$h_{r,p-b}(\delta_x)\left(T_p - T_a\right) = h_{c,b-a}(\delta_x)(T_b - T) + U_b(\delta_x)(T_b - T_a) \ \ldots (42)$$

As U_b is much smaller than U_t, $U_L \approx U_t$. Therefore, neglecting U_b and solving equation (42) for U_b gives

$$T_b = \frac{h_{r,p-b}T_p + h_{c,b-a}T}{h_{r,p-b} + h_{c,b-a}} \ \ldots (43)$$

Substituting equation (43) into equation (39) gives

$$T_a(U_L + h) = S + U_L T_a + hT \ \ldots (44)$$

$$h = h_{c,p-a} + \frac{1}{(1/h_{c,b-a}) + (1/h_{r,p-b})} \ \ldots (45)$$

Substituting equation (43) into equation (41) gives

$$hT_p = \left[\frac{m^{\circ}}{W}\right] c_p \left[\frac{dT}{dx}\right] + hT \ \ldots (46)$$

Finally, combining equation (44) and (46) gives

$$\left[\frac{m^{\circ}}{W}\right] c_p \left[\frac{dT}{dx}\right] = F'[S - U_L(T - T_a)] \ldots (47)$$

F' = Collector efficiency factor for air collectors, given by

$$F' = \frac{1/U_L}{(1/U_L) + (1/h)} = \frac{h}{h + U_L} \ \ldots (48)$$

The initial conditions of equation (47) are T = T$_i$ at x = 0. Therefore, the complete solution of equation (47) is

$$T = \left[\frac{S}{U_L} + T_a\right] + \frac{1}{U_L}[S - U_L(T_i - T_a)]exp\left[-\frac{U_L F'}{(m^{\circ}/W)c_p}x\right] \ldots (49)$$

This equation gives the temperature distribution of air in the duct. The temperature of the air at the outlet for the collector is obtained from Eq. (49), using **x = L** and considering **A$_c$ =WL**. Therefore,

$$T_o = T_i + \frac{1}{U_L}[S - U_L(T_i - T_a)]\left[1 - exp\left(-\frac{A_c U_L F'}{(m^{\circ})c_p}\right)\right] \ldots (50)$$

The energy gain by the air stream is then given by

$$\frac{Q_u}{W} = \left[\frac{m^{\circ}}{W}\right] c_p(T_o - T_i) = \frac{(m^{\circ}c_p)}{A_c U_L}[S - U_L(T_i - T_a)]\left[1 - exp\left(-\frac{A_c U_L F'}{(m^{\circ}c_p)}\right)\right] \ldots (51)$$

Using the equation for the heat removal factor, equation (51) gives

$$Q_u = A_c F_R[S - U_L(T_i - T_a)] \ \ldots (52)$$

Where S = (τα) I$_t$

Example: Estimate the outlet air temperature and efficiency of the collector shown in Figure (6) for the following collector specifications:

Collector width, W = 1.2 m.
Heat loss coefficient, UL = 6.5 W/m^2 -K.
Collector length, L = 4 m.
Emissivity of absorber plate, _p _ 0.92.
Depth of air channel, s = 15 mm
Emissivity of back plate, _b _ 0.92
Total insolation, I_t = 890 W/m^2
Mass flow rate of air _ 0.06 kg/s.
Ambient temperature, Ta = 15°C =288 K.
Inlet air temperature, Ti = 50°C = 323 K.
Effective (τα) = 0.90.

Assuming values for T_p = 340 K and T_b = 334 K.

Solution: the mean air temperature can be determined from

$$4(T_{m,\text{air}})^3 = (T_p + T_b)(T_p^2 + T_b^2)$$

$$T_{m,\text{air}} = \sqrt[3]{\frac{(T_p + T_b)(T_p^2 + T_b^2)}{4}} = \sqrt[3]{\frac{(340 + 334)(340^2 + 334^2)}{4}} = 337\,\text{K}$$

$$h_{r,p-g2} = \frac{\sigma(T_p + T_b)(T_p^2 + T_b^2)}{(1/\varepsilon_p) + (1/\varepsilon_b) - 1}$$

$$= \frac{(5.67 \times 10^{-8})(340 + 334)(340^2 + 334^2)}{(1/0.92) + (1/0.92) - 1}$$

$$= 7.395\,\text{W/m}^2\text{-K}$$

From $T_{m,\,air}$, the following properties of air

$$\mu = 2.051 \times 10^{-5}\ \text{kg/m-s}$$

$$k = 0.029\,\text{W/m-K}$$

$$c_p = 1008\,\text{J/kg-K}$$

From fluid mechanics the hydraulic diameter of the air channel is given by

$$D = 4\left(\frac{\text{Flow cross-sectional area}}{\text{Wetted perimeter}}\right) = 4\left(\frac{Ws}{2W}\right) = 2s$$

$$= 2 \times 0.015 = 0.03$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\dot{m} D}{A\mu} = \frac{0.06 \times 0.03}{(1.2 \times 0.015) \times 2.051 \times 10^{-5}}$$

$$= 4875.5$$

Therefore, the flow is turbulent, for which the following equation applies **Nu = 0.0158(Re)$^{0.8}$**. Since **Nu = (h_c D)/ k**, the convection heat transfer coefficient is given by

$$h_{c,p-a} = h_{c,b-a} = \left(\frac{k}{D}\right)0.0158(\text{Re})^{0.8}$$

$$= \left(\frac{0.029}{0.03}\right)0.0158(4875.5)^{0.8} = 13.625\,\text{W/m}^2\text{-K}$$

$$h = h_{c,p-a} + \frac{1}{(1/h_{c,b-a}) + (1/h_{r,p-b})} = 13.625 + \frac{1}{(1/13.625) + (1/7.395)}$$

$$= 18.4\,\text{W/m}^2\text{-K}$$

$$F' = \frac{h}{h + U_L} = \frac{18.4}{18.4 + 6.5} = 0.739$$

$$S = I_t(\tau\alpha) = 890 \times 0.9 = 801 \ W/m^2$$

$$T_o = T_i + \frac{1}{U_L}[S - U_L(T_i - T_a)]\left[1 - \exp\left(-\frac{A_c U_L F'}{\dot{m}c_p}\right)\right]$$

$$= 323 + \left(\frac{1}{6.5}\right)[801 - 6.5(323 - 288)]\left[1 - \exp\left(-\frac{(1.2 \times 4) \times 6.5 \times 0.739}{0.06 \times 1007}\right)\right]$$

$$= 351\,\mathrm{K}$$

Therefore, the average air temperature is ½(351 + 323) = 337 K, which is the same as the value assumed before. If there is a difference in the two mean values, an iteration is required.

$$F_R = \frac{\dot{m}c_p}{A_c U_L}\left\{1 - \exp\left[-\frac{U_L F' A_c}{\dot{m}c_p}\right]\right\}$$

$$= \frac{0.06 \times 1008}{(1.2 \times 4) \times 6.5}\left\{1 - \exp\left[-\frac{6.5 \times 0.739 \times (1.2 \times 4)}{0.06 \times 1008}\right]\right\} = 0.614$$

$$Q_u = A_c F_R[S - U_L(T_i - T_a)] = (1.2 \times 4) \times 0.614[801 - 6.5(323 - 288)]$$

$$= 1690\,\mathrm{W}$$

$$\eta = \frac{Q_u}{A_c I_t} = \frac{1690}{(1.2 \times 4) \times 890} = 0.396 = 39.6\%$$